P here denoting the variable point $\Lambda \cdot \Pi$, in which the fixed line Λ intersects the variable plane $\Pi = [lmnr]$.

20. All problems respecting *intersections of lines with planes*, &c., are resolved, with the help of the Fundamental Theorem (16) respecting the relation which exists between the anharmonic co-ordinates of point and plane, as easily by the present method, as by the known method of *quadriplanar* co-ordinates (10); and indeed, by the very *same mechanism*, of which it is therefore unnecessary here to speak.

But it may be proper to say a few words respecting the application of the anharmonic method to *Surfaces* (7); although here again the known mechanism of *calculation* may in great part be preserved unchanged, and only the *interpretations* need be *new*.

21. In general, it is easy to see (comp. 7) that, in the present method, as in older ones, the *order* of a curved surface is denoted by the *degree* of its *local equation*, $f(xyzw) = 0$; and that the *class* of the same surface is expressed, in like manner, by the degree of its *tangential equation*, $F(lmnr) = 0$: because the *former* degree (or dimension) determines the *number of points* (distinct or coincident, and real or imaginary), in which the surface, considered as a locus, is *intersected* by an arbitrary right *line;* while the *latter* degree determines the *number of planes* which can be drawn *through* an arbitrary right line, so as to touch the same surface, considered as an *envelope*. It may be added, that I find the *partial derivatives of each* of these two functions, f and F, to be proportional to the *co-ordinates* which enter as variables into the *other;* thus we may write

$$[D_x f,\ D_y f,\ D_z f,\ D_w f],$$

as the symbol (15) of the *tangent plane* to the *locus* f, at the point $(xyzw)$; and

$$(D_l F,\ D_m F,\ D_n F,\ D_r F),$$

as a symbol for the *point of contact* of the *envelope* F, with the plane $[lmnr]$: whence it is easy to conceive how problems respecting the *polar reciprocals* of *surfaces* are to be treated.

22. As a very simple *example*, the surface of the *second order* which passes through the *nine points*, above called ABCDEA$'$A$_2$C$'$C$_2$, is easily found to have for its *local* equation, $0 = f = xz - yw$; whence the co-ordinates of its tangent plane are, $l = D_x f = z$, $m = D_y f = -w$, $n = D_z f = x$, $r = D_w f = -y$, and its *tangential* equation is, therefore, $0 = F = ln - mr$, so that it is also a surface of the *second class*. In fact it is the *hyperboloid* on which the gauche quadrilateral ABCD is superscribed, and which passes also through the point E; and the known *double generation*, and *anharmonic properties*, of this surface, may easily be deduced from either of the foregoing forms of its anharmonic equation, whereof the first may (by 13, 15) be expressed as an equality between the anharmonic functions of two *pencils of planes*, in either of the two following ways:— (BC. AEDP) = (DA. BECP); (AB. CEDP) = (CD. BEAP).

ERRATUM.—In line 5 of Art. 12, for a read u.